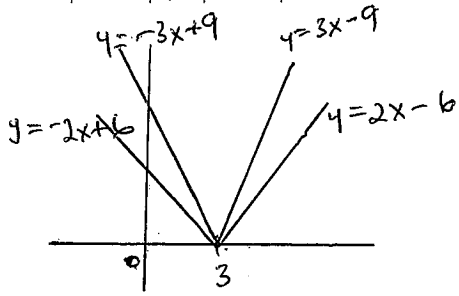
ST STEPHEN'S SCHOOL
Year 11 Specialist
Test 1, 2009

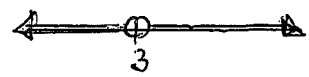
Monday March 9th 2009
Time: 50 mins.
Attempt all questions, show all working.
Calculators and notes are allowed.

1. (5 marks)

Solve $|2x-6| \geq |3x-9|$, giving your answer algebraically and on the number line.



$x \neq 3$



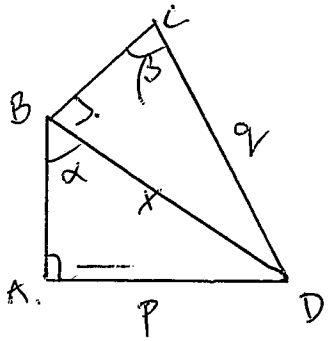
2. (3 marks)

Given that $|a+b| < |a| + |b|$ and a.b = k, where a, b and k are real numbers, what can you say about k?

a & b are of different sign.

$\therefore k < 0$.

3. (7 marks)
In quadrilateral ABCD, $\angle DAB = 90$, $\angle DBC = 90$, $\angle ABD = \alpha$, $\angle BCD = \beta$, AD = p, CD = q, prove that $q = \dfrac{p}{\sin\alpha . \sin\beta}$.



$\sin\alpha = \frac{p}{x}$ $\quad x = \frac{p}{\sin\alpha}$

$\sin\beta = \frac{x}{q}$ $\quad x = q\sin\beta$

$\therefore\ q\sin\beta = \frac{p}{\sin\alpha}$

$\therefore q = \frac{p}{\sin\alpha \sin\beta}$.

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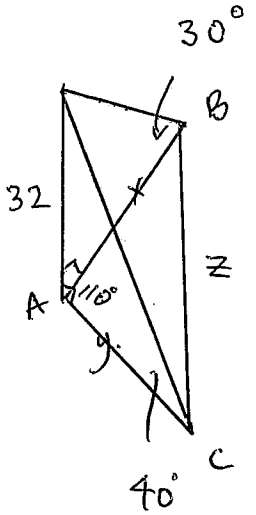
4. (8 marks)
Points A, B and C lie on horizontal ground with B due north of A and C on a bearing of 110 from A. A vertical flagpole of height 32m stands at A. From B and C the angles of elevation of the top of the flagpole are 30 and 40 respectively. Find the distance of B from C.



$\tan 30 = \frac{32}{x} \quad \therefore x = \frac{32}{\tan 30} = 55.43$

$\tan 40 = \frac{32}{y} \quad \therefore y = \frac{32}{\tan 40} = 38.14$

$z^2 = 55.43^2 + 38.14^2 - 2 \times 55.43 \times 38.14 \cos 110$

$= 5972.2$

$\therefore z = 77.28 \text{ m}$

5. (5 marks)
Prove that $\sin^2 60 + \cos^4 45 = \sin^2 30 + \cos^2 30$

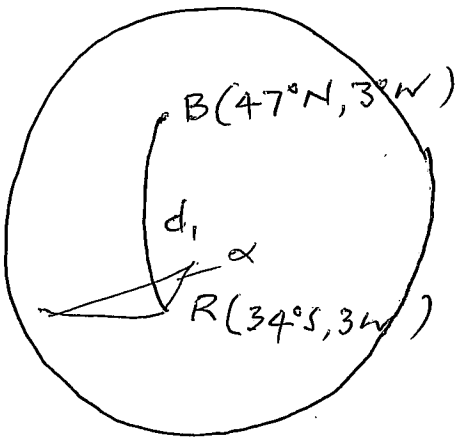
$\sin^2 60 + \cos^4 45 = \left(\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{\sqrt{2}}\right)^4 = \frac{3}{4} + \frac{1}{4} = 1$

$\sin^2 30 + \cos^2 30 = \left(\frac{1}{2}\right)^2 + \left(\frac{\sqrt{3}}{2}\right)^2 = \frac{1}{4} + \frac{3}{4} = 1$

} LHS = RHS

$\therefore \sin^2 60 + \cos^4 45 = \sin^2 30 + \cos^2 30.$

6. (8 marks)
The radius of the earth is 6350 km.
Find the distance a plane flies from Bassenthwaite (47N, 3W) to Rumencoco (34S, 3W).
If the plane then flies directly West for 17000 km, what will be its longitude (to the nearest degree)?



$d_1 = \frac{81}{360} \times 2\pi 6350 = \underline{8977 \text{ km}}$

$17000 = \frac{\alpha}{360} \cdot 2\pi 6350 \cos 34$

$\alpha = 185°$

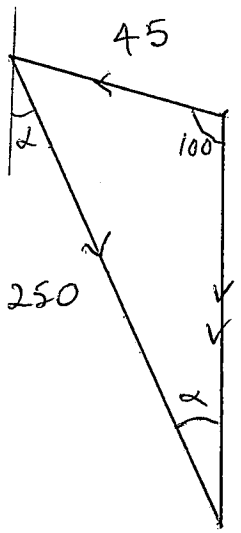
ie to 188°W! which is $\underline{172°E}$.

7. (7 marks)
In still air a plane can fly at 250 km/h. In what direction should it point to fly if it wants to travel due South when a 45km/h wind is blowing from 100 degrees.

$$\frac{\sin\alpha}{45} = \frac{\sin 100}{250}$$

$$\therefore \alpha = 10.2^{\circ}$$

$$\therefore \text{Dir}^{n} = 169.8^{\circ}$$



8. (7 marks)

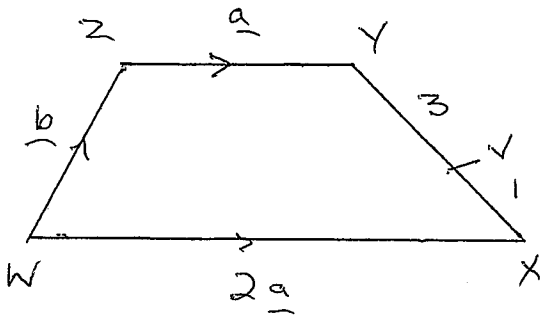
WXYZ is a trapezium with WX parallel to and twice as long as ZY. Given that $\overrightarrow{ZY} = \underline{a}$ and $\overrightarrow{WZ} = \underline{b}$, and V is a point on XY such that XV:VY = 1:3, express in terms of $\underline{a}$ a and/or $\underline{b}$

(a) $\overrightarrow{XY}$

(b) $\overrightarrow{XV}$

(c) $\overrightarrow{WV}$.

(d) $\overrightarrow{WM}$



(a) $\overrightarrow{XY} = -\underline{a} + \underline{b}$

(b) $\overrightarrow{XV} = \frac{1}{3}(-\underline{a} + \underline{b})$

(c) $\overrightarrow{WV} = 2\underline{a} + \frac{1}{3}(-\underline{a} + \underline{b})$

$= \frac{5}{3}\underline{a} + \frac{1}{3}\underline{b}$

(d) $\overrightarrow{WM} = \frac{1}{2}\overrightarrow{WY} = \frac{1}{2}(\underline{a} + \underline{b})$

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